

My high school was across the street from a Catholic-Chinese church in YinChuan, China. Its walls were built from white-ish bricks, which looked very funny to me for some reason. I looked at them often from the window during different classes. My math teacher was a survivor of the Chinese Revolution. She always wore long skirts, no matter in which season. She was older-looking, with a bit of a scary presence and odd teeth which gave her speech an unusual attitude and accent while teaching math. One day, in the same math class as usual, she showed up without looking at us and started writing massive problems on the gigantic blackboard. Boring. I started counting the bricks on the church, taking advantage of the window seat. The challenge of counting the bricks was enjoyable. I had to pretend to be attentive in the class while not losing count of the bricks outside the window.

As I counted to 98, my teacher said something about a tangent line, something which I had not heard of before. I focused back upon the blackboard, and at the same time tried to pin down the bricks so I could come back to counting afterwards. I saw a curve on the coordinate she drew with white chalk. Perhaps part of me was still with the white bricks from the church: the curve seemed to move itself off the board, starting with the rhythm of my counting, then accelerating. It went through the ceiling, the sky, then rose to kiss the sun. The line came back after light-years of traveling. It stayed on the blackboard. It took a nap. It dreamt about itself in a classroom on a hot summer day, with 34 pairs of eyes staring at it.

The fragments of sound from the bell in the church were accompanied by the summer wind. The tangent line woke up, but its world had been turned upside down after thousands of years. It started

soaring in downward, through the floors, the earth, to do something – maybe to kiss the moon? Nobody knows! It does not matter. The most intriguing fact was that, as the teacher said, it was a tangent formula. Then I woke up from this fantastical day dream. I saw a beautiful curve that looked nothing like the trees, the smiles, the buildings, the people. It was just a line with two small curves near the origin of the coordinate plane. It extended up and down forever. Why does a tangent line look like this? Where is it going exactly?

This memory of that soaring curve came back to me during the writing of this essay. Finally, it became clear that the sudden moment of seeing on the blackboard created a synesthetic glee between the mathematical symbols of $y = \tan(x)$ and the animated fantasy that the tangent curve invoked. It felt supremely magical. Is it possible to intentionally recreate this memorable experience? Does it make math more approachable if we pose questions like How does a tangent line feel? and introduce mathematical equations and methods with creative, intrinsic motivation?

To feel mathematics is to connect with its inherent beauty through our very senses in the way that we experience artworks. As the critic Viktor Shkolovsky (1917, p. 12) stated, “The purpose of art is to impart the sensation of things as they are perceived and not as they are known... Art is a way of experiencing the artfulness of an object; the object is not important.”

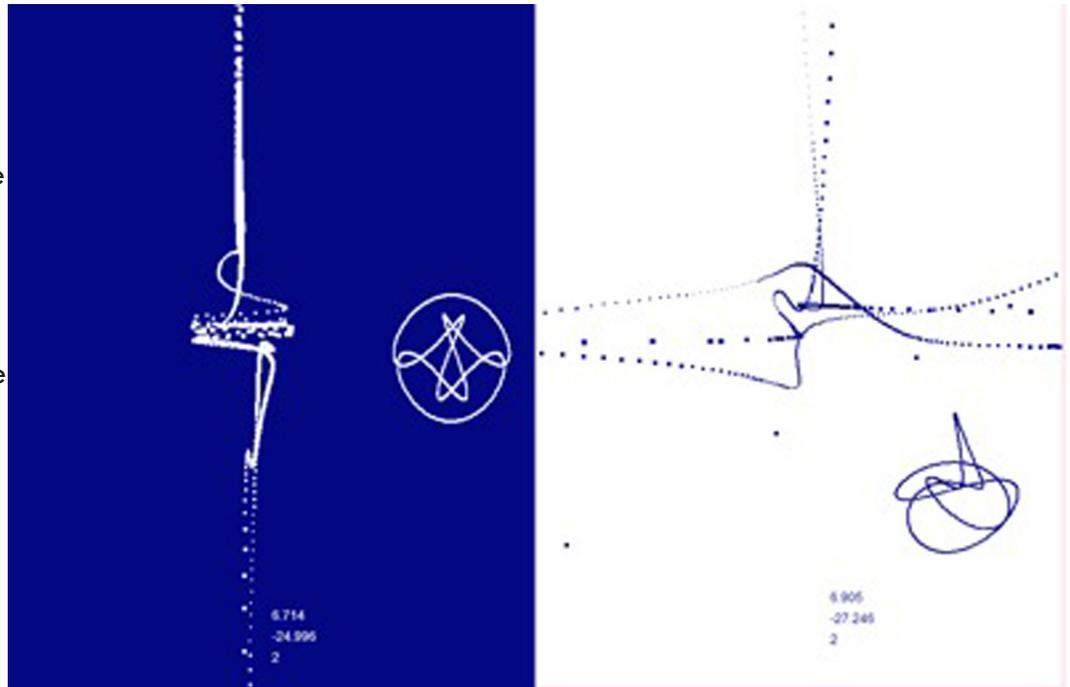
For me, this dance piece “On Line” by Anne Teresa De Keersmaeker feels like mathematics. The movements visualize her intrinsic feelings of geometry and the internalized rhythm of mathematical operation. She danced periodically on a flat surface of untouched

sand, drawing an oscillating pattern, and revealing a sense of mathematical urgency which is symbolized and persistent. Accompanied by the soundscape of Steve Riche's Violin Phases, "On Line" amplified the simulation of what is it like to perceive the growth of repetitive forms systematically and aesthetically. For me, the synesthesia of this dance is to emotionally feel a formative frequency which projects a vague sense of mathematical processes while experiencing this dance. As Anne mentioned in an interview," I am obsessed by structures. But the most beautiful experience is to see such a construction generating something intangible, elusive –an emotion" (Roy, 2009). Perhaps mathematics cannot provoke vivid sensations in everyone; however, is it possible to experiment with these tactile mathe-

tical-sensory transformations by exploring how we relate to nature? Alfred North Whitehead (as cited in Woodhouse, 2006) emphasized the concept that mathematics could unlock the alternating rhythms of repetition and difference in nature that constitute the periodicity of life. Yes, we inevitably feel the sway of nature – day and night, the ebb and flow of tides, and the breathing of ourselves and the person close by. The attempt to imagine and simulate a scenario that consists of mathematics, art, life and nature seems sublime. This endeavor or attempt might encourage students' inner curiosities of life, including their interests in art or mathematics.

The name Reasonless Math presented itself to me after I had spent months writing artist statements for various production opportunities. This term captures the underlying exploratory qualities of art-making while emphasizing navigation within the context of mathematics. It clarifies that the kind of mathematical visualization I am practicing is neither in the interest of beautifying math nor in displaying the fascinating and beautiful visuals it can produce. Rather, I wish to show the uncertainty of exploring equations, the unknown

aspects of algebraically and visually operating frequencies, the playfulness of programming code to introduce fluid time into otherwise static equations. Those purposeless and reasonless mathematical operations to me are worthwhile; they have the most potential to generate evocative visual studies.



Still from Sealed. To see the visualization in motion, please visit <http://vokeart.org/zhu>.

Sealed is an example of a Reasonless Math experiment. It is based upon the mental image of the soaring tangent line, although the two visualizations are subject to the following parametric [1] equation:

$$y = [r*(1 + \cos(1*(time1) + time2))] * \sin(3*(time1 + time2));$$

$$x = [r*(1 + \cos(1*(time1) + time2))] * \cos(3*(time1 + time2));$$

The structure of Sealed indicates only one difference between the x and y functions: y has the multiplication of sine, while x has the multiplication of cosine. In analytical geometry, one way to express a perfect circle in a coordinate system is to have the point P(x,y) where $x = \sin(t)$, $y = \cos(t)$. Therefore, in order to generate a circle-like visualization (static or mobile) within the parametric functions of Sealed, the structures of x and y were designed by multiplying by sine in y and by cosine in x. The figure on the lower right displays the result of this operation.

Despite the element of complexity in both x and y functions, the soaring figure on the left is the visual result of multiplying this pair of parametric equations by the tangent function in each x and y axis, and doing so over the same parametric combination of speed and time (time1) + (time2). That is, the soaring figure is the result of multiplying the circular motion on the lower right by the tangent function. By knowing the structures of the two equations, we see the layers of visual relationships among the motion in movements and the geometrical dynamics. Consequently, their juxtaposition highlights properties that the tangent function holds over time (in the shared rhythms of the two forms) and illustrates the distinct properties of each form's mathematical transformation (in the visible differences between the two forms). The white visualization which accompanies this one will be discussed below.

Typically, the strategy of translating an equation into a visual expression is called graphing the equation (Von Seggern, 1993); however, the term visualizing might be more appropriate than graphing to discuss this concept of translating a written formula into a picture. The former denotes plotting the values of a formula on a coordinate system, while the latter emphasizes the act of visually communicating the information embedded in that formula.

Analytical geometry transforms mathematical elements and relationships from equations to visual composition. At the beginning of my practice, I toyed with the structures of parametric combinations, which is a considerably elementary practice as far as the mathematical method is concerned. It deals only with the arrangements of functions and changing numbers for the sake of creating variations in variables. Later, I begin to explore derivatives.

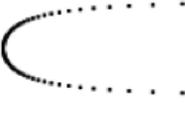
The problem of finding the tangent line to a curve and the problem of finding the velocity of an object both involve finding the same type of limit. This special type of limit is called a derivative and we will see that it can be interpreted as a rate of change in any of the sciences or engineering (Stewart, 2010, p. 143).

Visualizing simple derivatives of trigonometric equations adds another exploratory dimension to Reasonless Math. It elicits questions about what kinds of motion and visual quality a complex parametric motion will display and whether its derivatives are being calculated for the pure reason of visualization. The following diagram demonstrates the artfulness of this transformation from equation to coordinate visualization.

	+	-	×	÷
Original $\begin{pmatrix} x = \sin \theta \\ y = \cos \theta \end{pmatrix}$	$\begin{pmatrix} x = \sin \theta + \cos \theta \\ y = \cos \theta \end{pmatrix}$	$\begin{pmatrix} x = \sin \theta - \cos \theta \\ y = \cos \theta \end{pmatrix}$	$\begin{pmatrix} x = \sin \theta \times \cos \theta \\ y = \cos \theta \end{pmatrix}$	$\begin{pmatrix} x = \sin \theta \div \cos \theta \\ y = \cos \theta \end{pmatrix}$
1st derivative $\begin{pmatrix} x = \sin \theta \\ y = \cos \theta \end{pmatrix}'$	$\begin{pmatrix} x = \sin \theta + \cos \theta \\ y = \cos \theta \end{pmatrix}'$	$\begin{pmatrix} x = \sin \theta - \cos \theta \\ y = \cos \theta \end{pmatrix}'$	$\begin{pmatrix} x = \sin \theta \times \cos \theta \\ y = \cos \theta \end{pmatrix}'$	$\begin{pmatrix} x = \sin \theta \div \cos \theta \\ y = \cos \theta \end{pmatrix}'$
2nd derivative $\begin{pmatrix} x = \sin \theta \\ y = \cos \theta \end{pmatrix}''$	$\begin{pmatrix} x = \sin \theta + \cos \theta \\ y = \cos \theta \end{pmatrix}''$	$\begin{pmatrix} x = \sin \theta - \cos \theta \\ y = \cos \theta \end{pmatrix}''$	$\begin{pmatrix} x = \sin \theta \times \cos \theta \\ y = \cos \theta \end{pmatrix}''$	$\begin{pmatrix} x = \sin \theta \div \cos \theta \\ y = \cos \theta \end{pmatrix}''$

Mathematical visualization is a broader general term when applied to analytical geometry. It may describe the study of pure math, of data mining for statistics, or of waves and oscillation in physics [2]. Within the visualization, the symbol P(x, y) indicates a point or sequence of points, the position of which is defined by the values of x and y. P(x, y) is sometimes written as P(x, f(x)), where y = f(x). This emphasizes that x is input into a function, f, which results in y. As a visible point, P(x, f(x)) is a visualization of its function.

The table shows configurations in the realm of analytical geometry. To avoid complication, the coordinate lines are hidden. The first column shows the visualization of two simple formulas sin(θ), cos(θ), as points P(sin(θ), cos(θ)). Note that the visualization of these formulas is a circle. Each successive column shows the visualization of a different operation being executed on sin(θ) and cos(θ). Each descending row sequentially shows the derivatives of the equations in the row above.

	+	-	×	÷
Original				
1st derivative				
2nd derivative				

The purpose of the above diagram is neither to investigate the mathematical operation of formulas nor to display the graphical curves. It illustrates the embodiment of gradual aesthetic and geometric relations among the matrix of algebraic operations, trigonometric functions, and derivative transformations. It allows the viewer to see what a “+,” “-,” “×,” or “÷” looks like. Apart from the pragmatic reasons for visualizing mathematics, those curves are inherent in each formula or series of formulas. Likewise, the white visualization shown earlier visualizes the derivative of the blue visualization it accompanies, illuminating how such an operation transforms that more complex parametric equation.

The active reading of the graphs is an exercise of transforming cognitive reasoning to visually thinking (Walker, Winner, Hetland, Simmons, & Goldsmith, 2011). Whether or not it is a necessary and valuable exercise, for me it has served to transcend the methodical rules of derivatives and algebra into a poetic creation. It recalls the joy of creating experienced in childhood, where the manipulation and recombination of simple elements could transform a known substance into something beautiful and new. The philosophy and the story of analytical geometry might encourage young students or interested professionals to creatively experiment with mathematical formulas and, at the same time, to appreciate the experience of creating art. Reasonless Math emphasizes that certain content in mathematics education can

be experimentally treated as an art practice, transforming students into artists working in the medium of mathematics to create their own narratives of where information and knowledge meet.

Mathematics, rightly viewed, possesses not only truth, but supreme beauty—a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show (Russell, 1919).

Within the realm of love for mathematics and art held by the great thinker Bertrand Russell, it seems as though math and art are spiritually connected, despite how rarely their fundamental and factual connections are remarked upon. For myself, I find it unnecessary to categorize the connections or essential differences between art and mathematics. Still, the opportunity to experiment with a kind of personalized integration driven from each individual’s fondness for the two subjects should not be overlooked. In this way, the creative output can manifest organically in whatever field inspires: art making, education, visual design, or pure mathematical exploration.

[1]Parametric equations/formulas indicate the relationship between horizontal distance and time. The concept was developed by Galileo, who attempted to investigate curvilinear motion over time.

[2] For more information please read this short introduction by Stewart Calculus: <http://www.stewartcalculus.com/data/ESSENTIAL%20CALCULUS/upfiles/ess-reviewofanalgeom.pdf>

[3] Analytical geometry was developed by Renee Descartes (1641/1986), who thought that abstract

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